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Quantum Thermal Effect of Dirac Particles in a Nonuniformly Rectilinearly Accelerating Black Hole With Electric Charge, Magnetic Charge, and Cosmological Constant

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The Hawking radiation of Dirac particles in an arbitrarily rectilinearly accelerating Kinnersley black hole with electromagnetic charge and cosmological constant is investigated by using the generalized tortoise coordinate transformation. Both the location and the temperature of the event horizon depend on the time and the polar angle. The Hawking thermal radiation spectrum of Dirac particles is also derived.

KEY WORDS: Hawking effect; Dirac equation; nonstationary black hole; generalized tortoise coordinate transformation.

1. INTRODUCTION

An important subject of black hole physics is to reveal the thermal properties of various black holes (Hawking, 1974, 1975). The last decade has witnessed much progress in investigating the thermal properties of scalar fields or Dirac particles in the stationary axisymmetry black holes (Damour and Ruffini, 1976; Liu and Xu, 1980; Xu, 1983; Xu and Shen, 1982; Zhao *et al.*, 1981). In the study of the Hawking evaporation of the nonstationary black holes, the method of the generalized tortoise coordinate transformation (GTCT) suggested by Zhao and Dai (1991) has been applied to investigate the Hawking thermal radiation of scalar particles in some nonuniformly accelerating black holes (Luo and Zhao, 1993; Zhao *et al.*, 1995; Zhu *et al.*, 1994a) and in the nonuniformly accelerating Kerr black hole (Sun *et al.*, 1995a,b; Wu and Zhao, 1993).

However, it is very difficult to investigate the quantum thermal effect of Dirac particles in the nonstationary black hole. The difficulty lies in the nonseparability of

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the Chandrasekhar–Dirac equation (Chandrasekhar, 1983; Page, 1976) in the most general space-times. The Hawking radiation of Dirac particles in some nonstatic black holes has so far been studied in Li and Zhao (1993), Ma and Yang (1993), Zhang *et al.* (1999), and Zhu *et al.* (1994b).

In this paper, we deal with the Hawking effect of Dirac particles in a nonspherically symmetric and nonstationary Kinnersley black hole with electric charge, magnetic charge, and cosmological constant (Kinnersley, 1969; Wang and Tang, 1986). By making use of the GTCT method, we obtain the equation which determines the event horizon of the Kinnersley black hole. The event horizon equation derived by the limiting form of Dirac equation near the event horizon is exactly the same as those given by the null hypersurface which is not spherically symmetric (Luo and Zhao, 1993; Zhao *et al.*, 1995; Zhu *et al.*, 1994a). Then we turn to the second order form of the Dirac equation. With the aid of a GTCT, we adjust the temperature parameter in order that each component of Dirac spinors satisfies a simple wave equation after being taken limits approaching the event horizon.

We show that both the shape and the Hawking temperature of the event horizon of Kinnersley black hole depend on not only the time, but also on the angle. The location and the temperature coincide with those obtained by investigating the Hawking effect of Klein–Gordon particles in the accelerating Kinnersley black hole (Luo and Zhao, 1993; Zhao *et al.*, 1995; Zhu *et al.*, 1994a).

2. DIRAC EQUATION

The metric of a nonuniformly rectilinearly accelerating Kinnersley black hole with electric charge Q, magnetic charge P, and cosmological constant Λ is given in the advanced Eddington–Finkelstein coordinate system by Kinnersley (1969) and Wang and Tang (1986).

$$ds^2 = 2dv(G dv - dr - r^2 f\, d\theta) - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{1}$$

where $2G = 1 - \frac{2M}{r} + \frac{Q^2+P^2}{r^2} - 4a\cos\theta\frac{Q^2+P^2}{r} - 2ar\cos\theta - r^2 f^2 - \frac{\Lambda}{3}r^4$, $f = -a\sin\theta$. In the above, the parameter $a = a(v)$ is the magnitude of acceleration, the mass $M(v)$, and the charges $Q(v)$, $P(v)$ of the hole are functions of time v.

We choose a complex null-tetrad $\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}}\}$ such that $\boldsymbol{l}\cdot\boldsymbol{n} = -\boldsymbol{m}\cdot\bar{\boldsymbol{m}} = 1$. Thus the covariant one-forms can be written as

$$\begin{aligned} \boldsymbol{l} &= dv, \qquad \boldsymbol{n} = G dv - dr - r^2 f\, d\theta, \\ \boldsymbol{m} &= \frac{-r}{\sqrt{2}}(d\theta + i\sin\theta\, d\varphi), \qquad \bar{\boldsymbol{m}} = \frac{-r}{\sqrt{2}}(d\theta - i\sin\theta\, d\varphi). \end{aligned} \tag{2}$$

and their corresponding directional derivatives are

$$D = -\frac{\partial}{\partial r}, \qquad \Delta = \frac{\partial}{\partial v} + G\frac{\partial}{\partial r},$$

$$\delta = \frac{1}{\sqrt{2}r}\left(-r^2 f \frac{\partial}{\partial r} + \frac{\partial}{\partial \theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial \varphi}\right),$$
$$\overline{\delta} = \frac{1}{\sqrt{2}r}\left(-r^2 f \frac{\partial}{\partial r} + \frac{\partial}{\partial \theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial \varphi}\right). \tag{3}$$

Inserting for the following relations among the Newman–Penrose (1962) spin-coefficients[3]

$$\epsilon - \rho = -\frac{1}{r}, \qquad \tilde{\pi} - \alpha = \frac{\cot\theta}{2\sqrt{2}r} - \sqrt{2}f,$$
$$\mu - \gamma = \frac{G}{r} + \frac{G_{,r}}{2}, \qquad \beta - \tau = \frac{\cot\theta}{2\sqrt{2}r} - \frac{f}{\sqrt{2}}, \tag{4}$$

into the spinor form of the coupled Chandrasekhar–Dirac equation (Chandrasekhar, 1983; Page, 1976), which describes the dynamic behavior of spin-1/2 particles, namely

$$(D + \epsilon - \rho)F_1 + (\overline{\delta} + \tilde{\pi} - \alpha)F_2 = \frac{i\mu_0}{\sqrt{2}}G_1,$$
$$(\Delta + \mu - \gamma)F_2 + (\delta + \beta - \tau)F_1 = \frac{i\mu_0}{\sqrt{2}}G_2,$$
$$(D + \epsilon^* - \rho^*)G_2 - (\delta + \tilde{\pi}^* - \alpha^*)G_1 = \frac{i\mu_0}{\sqrt{2}}F_2,$$
$$(\Delta + \mu^* - \gamma^*)G_1 - (\overline{\delta} + \beta^* - \tau^*)G_2 = \frac{i\mu_0}{\sqrt{2}}F_1, \tag{5}$$

where μ_0 is the mass of Dirac particles, one obtains

$$-\mathcal{D}_1 F_1 + \frac{1}{\sqrt{2}r}(\mathcal{L} - r^2 f \mathcal{D}_2)F_2 = \frac{i\mu_0}{\sqrt{2}}G_1,$$
$$\left(\frac{\partial}{\partial v} + G\mathcal{D}_1 + G_{,r}/2\right)F_2 + \frac{1}{\sqrt{2}r}(\mathcal{L}^\dagger - r^2 f \mathcal{D}_1)F_1 = \frac{i\mu_0}{\sqrt{2}}G_2,$$
$$-\mathcal{D}_1 G_2 - \frac{1}{\sqrt{2}r}(\mathcal{L}^\dagger - r^2 f \mathcal{D}_2)G_1 = \frac{i\mu_0}{\sqrt{2}}F_2,$$
$$\left(\frac{\partial}{\partial v} + G\mathcal{D}_1 + G_{,r}/2\right)G_1 - \frac{1}{\sqrt{2}r}(\mathcal{L} - r^2 f \mathcal{D}_1)G_2 = \frac{i\mu_0}{\sqrt{2}}F_1, \tag{6}$$

in which we have defined operators

$$\mathcal{D}_n = \frac{\partial}{\partial r} + \frac{n}{r}, \quad \mathcal{L} = \frac{\partial}{\partial \theta} + \frac{1}{2}\cot\theta - \frac{i}{\sin\theta\,\partial\varphi},$$
$$\mathcal{L}^\dagger = \frac{\partial}{\partial \theta} + \frac{1}{2}\cot\theta + \frac{i}{\sin\theta}\frac{\partial}{\partial \varphi}.$$

[3] Here and hereafter, we denote $G_{,r} = dG/dr$, etc.

By substituting

$$F_1 = \frac{1}{\sqrt{2}r} P_1, \quad F_2 = P_2, \quad G_1 = Q_1, \quad G_2 = \frac{1}{\sqrt{2}r} Q_2,$$

into Eq. (6), they have the form

$$\begin{aligned} -\mathcal{D}_0 P_1 + (\mathcal{L} - r^2 f \mathcal{D}_2) P_2 &= i\mu_0 r Q_1, \\ r^2\left(2\frac{\partial}{\partial v} + 2G\mathcal{D}_1 + G_{,r}\right) P_2 + (\mathcal{L}^\dagger - r^2 f \mathcal{D}_0) P_1 &= i\mu_0 r Q_2, \\ -\mathcal{D}_0 Q_2 - (\mathcal{L}^\dagger - r^2 f \mathcal{D}_2) Q_1 &= i\mu_0 r P_2, \\ r^2\left(2\frac{\partial}{\partial v} + 2G\mathcal{D}_1 + G_{,r}\right) Q_1 - (\mathcal{L} - r^2 f \mathcal{D}_0) Q_2 &= i\mu_0 r P_1. \end{aligned} \tag{7}$$

3. EVENT HORIZON

An apparent fact is that the Chandrasekhar–Dirac equation (7) could be satisfied by identifying Q_1, Q_2 with P_2^*, $-P_1^*$, respectively. So one may deal with a pair of components P_1, P_2 only. Although Eq. (7) cannot be decoupled, to deal with the problem of Hawking radiation, one may concern about the behavior of Eq. (7) near the horizon only. As the space-time we consider at present has a symmetry about φ-axis, we can introduce the generalized tortoise coordinate transformation (Zhao and Dai, 1991)

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa} \ln[r - r_H(v, \theta)], \\ v_* &= v - v_0, \qquad \theta_* = \theta - \theta_0, \end{aligned} \tag{8}$$

where r_H is the location of the event horizon, κ is an adjustable parameter and is unchanged under tortoise transformation. Both parameters v_0 and θ_0 are arbitrary constants. From formula (8), we can deduce some useful relations for the derivatives as follows:

$$\begin{aligned} \frac{\partial}{\partial r} &= \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial v} &= \frac{\partial}{\partial v_*} - \frac{r_{H,v}}{2\kappa(r - r_H)} \frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial \theta} &= \frac{\partial}{\partial \theta_*} - \frac{r_{H,\theta}}{2\kappa(r - r_H)} \frac{\partial}{\partial r_*}. \end{aligned}$$

Under the transformation (8), Eq. (7) with regards to (P_1, P_2) can be reduced to the following limiting form near the event horizon[4]

$$\begin{aligned} \frac{\partial}{\partial r_*}P_1 + \left(r_{H,\theta} + r_H^2 f\right)\frac{\partial}{\partial r_*}P_2 &= 0, \\ -\left(r_{H,\theta} + r_H^2 f\right)\frac{\partial}{\partial r_*}P_1 + 2r_H^2(G - r_{H,v})\frac{\partial}{\partial r_*}P_2 &= 0, \end{aligned} \tag{9}$$

after being taken limits $r \to r_H(v_0, \theta_0)$, $v \to v_0$, and $\theta \to \theta_0$. A similar form holds for Q_1, Q_2.

If the derivatives $\frac{\partial}{\partial r_*}P_1$ and $\frac{\partial}{\partial r_*}P_2$ in Eq. (9) do not be equal to zero, the existence condition of nontrial solutions for P_1 and P_2 is that the determinant of Eq. (9) vanishes, which gives the following equation to determine the location of horizon

$$2G - 2r_{H,v} + r_H^2 f^2 + 2 f r_{H,\theta} + \frac{r_{H,\theta}^2}{r_H^2} = 0. \tag{10}$$

The event horizon equation (10) can be inferred from the null hypersurface condition, $g^{ij}\partial_i F \partial_j F = 0$, and $F(v, r, \theta) = 0$, namely $r = r(v, \theta)$. The location of the event horizon is in accord with that obtained in the case of discussion about the thermal effect of Klein–Gordon particles in the same space-time (Luo and Zhao, 1993; Zhao *et al.*, 1995; Zhu *et al.*, 1994a). It follows that r_H depends not only on v, but also on θ. So the location of the event horizon and the shape of the black hole change with time.

4. HAWKING TEMPERATURE

To investigate the Hawking radiation of spin-1/2 particles, one may only deal with the behavior of P_1, P_2 components of Dirac equation near the event horizon because one can set

$$Q_2 = -P_1^*, \qquad Q_1 = P_2^*. \tag{11}$$

A direct calculation gives the second-order form of Dirac equation for the two-component spinor (P_1, P_2)

$$\begin{aligned} &\left[r^2\left(2\frac{\partial}{\partial v} + 2G\mathcal{D}_0 + G_{,r}\right)\mathcal{D}_0 + (\mathcal{L} - r^2 f\mathcal{D}_{-1})(\mathcal{L}^\dagger - r^2 f\mathcal{D}_0)\right]P_1 \\ &\quad + r^2\left[-\left(2\frac{\partial}{\partial v} + 2G\mathcal{D}_0 + G_{,r}\right)(\mathcal{L} - r^2 f\mathcal{D}_2)\right. \\ &\quad \left. + (\mathcal{L} - r^2 f\mathcal{D}_1)\left(2\frac{\partial}{\partial v} + 2G\mathcal{D}_1 + G_{,r}\right)\right]P_2 = \mu_0^2 r^2 P_1, \end{aligned} \tag{12}$$

[4] Throughout the paper, we make a convention that all coefficients in the front of each derivatives term take values at the event horizon when a GTCT is made and followed by taking limits approaching the event horizon.

and

$$\left[r^2\mathcal{D}_1\left(2\frac{\partial}{\partial v}+2G\mathcal{D}_1+G_{,r}\right)+(\mathcal{L}^\dagger-r^2f\mathcal{D}_1)(\mathcal{L}-r^2f\mathcal{D}_2)\right]P_2$$
$$+[\mathcal{D}_{-1}(\mathcal{L}^\dagger-r^2f\mathcal{D}_0)-(\mathcal{L}^\dagger-r^2f\mathcal{D}_1)\mathcal{D}_0]P_1=\mu_0^2r^2P_2. \tag{13}$$

Given the GTCT in Eq. (8) and after some tedious calculations, the limiting form of Eqs. (12) and (13), when r approaches $r_H(v_0,\theta_0)$, v goes to v_0 and θ goes to θ_0, reads

$$\left[\frac{A}{2\kappa}+2r_H^2(2G-r_{H,v})+2r_H^4f^2+2fr_{H,\theta}r_H^2\right]\frac{\partial^2}{\partial r_*^2}P_1+2r_H^2\frac{\partial^2}{\partial r_*\partial v_*}P_1$$
$$-2\big(fr_H^2+r_{H,\theta}\big)\frac{\partial^2}{\partial r_*\partial\theta_*}P_1+\big[-A+r_H^2G_{,r}+r_H^3f^2-r_H^2f\ \cot\theta_0$$
$$-r_H^2f_{,\theta}-(r_Hf+\cot\theta_0)r_{H,\theta}-r_{H,\theta\theta}\big]\frac{\partial}{\partial r_*}P_1+\ 2r_H^2\bigg[r_H^2f_{,v}$$
$$+G_{,\theta}-\frac{Gr_{H,\theta}}{r_H}-r_H^2f\left(G_{,r}+\frac{r_{H,v}-2G}{r_H}\right)\bigg]\frac{\partial}{\partial r_*}P_2=0, \tag{14}$$

and

$$\left[\frac{A}{2\kappa}+2r_H^2(2G-r_{H,v})+2r_H^4f^2+2fr_{H,\theta}r_H^2\right]\frac{\partial^2}{\partial r_*^2}P_2+2r_H^2\frac{\partial^2}{\partial r_*\partial v_*}P_2$$
$$-2\big(fr_H^2+r_{H,\theta}\big)\frac{\partial^2}{\partial r_*\partial\theta_*}P_2+\big[-A+3r_H^2G_{,r}+\ 2r_H(2G-r_{H,v})$$
$$+5r_H^3f^2-r_H^2f_{,\theta}-r_H^2f\ \cot\theta_0+(3fr_H-\cot\theta_0)r_{H,\theta}-r_{H,\theta\theta}\big]$$
$$\times\frac{\partial}{\partial r_*}P_2+\frac{r_{H,\theta}}{r_H}\frac{\partial}{\partial r_*}P_1=0. \tag{15}$$

With the aid of the event horizon equation (10), we know that the coefficient A is an infinite limit of $\frac{0}{0}$ type. By use of the L' Hôspital rule, we get the following result

$$A=\lim_{r\to r_H(v_0,\theta_0)}\frac{2r^2(G-r_{H,v})+r^4f^2+2fr^2r_{H,\theta}+r_{H,\theta}^2}{r-r_H}$$
$$=2r_H^2G_{,r}+4r_H(G-r_{H,v})+4r_H^3f^2+4fr_Hr_{H,\theta}$$
$$=2r_H^2G_{,r}+2r_H^3f^2-\frac{2r_{H,\theta}^2}{r_H}. \tag{16}$$

Now let us select the adjustable parameter κ in Eqs. (14) and (15) such that

$$\begin{aligned} r_H^2 &\equiv \frac{A}{2\kappa} + 2r_H^2(2G - r_{H,v}) + 2r_H^4 f^2 + 2fr_H^2 r_{H,\theta} \\ &= \frac{r_H^3 G_{,r} + r_H^4 f^2 - r_{H,\theta}^2}{\kappa r_H} + 2Gr_H^2 + r_H^4 f^2 - r_{H,\theta}^2, \end{aligned} \tag{17}$$

which means the temperature of the horizon is

$$\kappa = \frac{r_H^2 G_{,r} + r_H^3 f^2 - r_{H,\theta}^2 / r_H}{r_H^2(1-2G) - r_H^4 f^2 + r_{H,\theta}^2}. \tag{18}$$

Such a parameter adjustment can make Eqs. (14) and (15) reduce to

$$\begin{aligned} &\frac{\partial^2}{\partial r_*^2}P_1 + 2\frac{\partial^2}{\partial r_* \partial v_*}P_1 - 2\left(f + \frac{r_{H,\theta}}{r_H^2}\right)\frac{\partial^2}{\partial r_* \partial \theta_*}P_1 + \Bigg[r_H f^2 - G_{,r} \\ &\quad - f_{,\theta} - f\cot\theta_0 - (r_H f + \cot\theta_0)\frac{r_{H,\theta}}{r_H^2} + \frac{2r_{H,\theta}^2}{r_H^3} - \frac{r_{H,\theta\theta}}{r_H^2}\Bigg]\frac{\partial}{\partial r_*}P_1 \\ &\quad + 2\left[G_{,\theta} + r_H^2 f_{,v} - \frac{G r_{H,\theta}}{r_H} - r_H^2 f\left(G_{,r} + \frac{r_{H,v} - 2G}{r_H}\right)\right]\frac{\partial}{\partial r_*}P_2 = 0, \end{aligned} \tag{19}$$

and

$$\begin{aligned} &\frac{\partial^2}{\partial r_*^2}P_2 + 2\frac{\partial^2}{\partial r_* \partial v_*}P_2 - 2\left(f + \frac{r_{H,\theta}}{r_H^2}\right)\frac{\partial^2}{\partial r_* \partial \theta_*}P_2 + \Bigg[5r_H f^2 + G_{,r} \\ &\quad + \frac{4G - 2r_{H,v}}{r_H} - f_{,\theta} - f\cot\theta_0 + (3fr_H - \cot\theta_0)\frac{r_{H,\theta}}{r_H^2} \\ &\quad + \frac{2r_{H,\theta}^2}{r_H^3} - \frac{r_{H,\theta\theta}}{r_H^2}\Bigg]\frac{\partial}{\partial r_*}P_2 + \frac{r_{H,\theta}}{r_H^3}\frac{\partial}{\partial r_*}P_1 = 0. \end{aligned} \tag{20}$$

Using Eq. (9), Eqs. (19) and (20) can be recast into the following standard wave equation near the horizon in an united form

$$\frac{\partial^2}{\partial r_*^2}\Psi + 2\frac{\partial^2}{\partial r_* \partial v_*}\Psi - 2C_1\frac{\partial^2}{\partial r_* \partial \theta_*}\Psi + 2C_2\frac{\partial}{\partial r_*}\Psi = 0, \tag{21}$$

where C_1, C_2 will all be regarded as finite real constants,

$$C_1 = f + \frac{r_{H,\theta}}{r_H^2},$$

$$2C_2 = -r_H f^2 - G_{,r} - f_{,\theta} - f\cot\theta_0 - (r_H f + \cot\theta_0)\frac{r_{H,\theta}}{r_H^2} + \frac{2r_{H,\theta}^2}{r_H^3} - \frac{r_{H,\theta\theta}}{r_H^2}$$

$$-\frac{r_H^2 f + r_{H,\theta}}{(G - r_{H,v})r_H^3}\left[Gr_{H,\theta} - r_H G_{,\theta} - r_H^3 f_{,v} + r_H^2 f(G_{,r} r_H + r_{H,v} - 2G)\right]$$

for $\Psi = P_1$, and

$$2C_2 = 3r_H f^2 - f_{,\theta} - f \cot\theta_0 + G_{,r} + \frac{4G - 2r_{H,v}}{r_H}$$
$$+ (2fr_H - \cot\theta_0)\frac{r_{H,\theta}}{r_H^2} + \frac{r_{H,\theta}^2}{r_H^3} - \frac{r_{H,\theta\theta}}{r_H^2}$$

for $\Psi = P_2$.

5. THERMAL RADIATION SPECTRUM

Now separating variable as follows

$$\Psi = R(r_*)\Theta(\theta_*)\, e^{-i\omega v_* + im\varphi}$$

and substituting this into Eq. (21), one gets

$$\begin{aligned} \Theta' &= \lambda\Theta, \\ R' &= 2(i\omega - C_0)R, \end{aligned} \tag{22}$$

where λ is a real constant introduced in the separation variables, $C_0 = C_2 - \lambda C_1$. The solutions are

$$\begin{aligned} \Theta &= e^{\lambda\theta_*}, \\ R &= e^{2(i\omega - C_0)r_*}; R_0. \end{aligned} \tag{23}$$

The ingoing wave and the outgoing wave to Eq. (21) are

$$\begin{aligned} \Psi_{\text{in}} &= e^{-i\omega v_* + im\varphi + \lambda\theta_*}, \\ \Psi_{\text{out}} &= e^{-i\omega v_* + im\varphi + \lambda\theta_*}\, e^{2(i\omega - C_0)r_*}, \quad (r > r_H). \end{aligned} \tag{24}$$

Near the event horizon, we have

$$r_* \sim \frac{1}{2\kappa}\ \ln(r - r_H).$$

Clearly, the outgoing wave $\Psi_{\text{out}}(r > r_H)$ is not analytic at the event horizon $r = r_H$, but can be analytically extended from the outside of the hole into the inside of the hole through the lower complex r-plane

$$(r - r_H) \to (r_H - r)\, e^{-i\pi}$$

to

$$\tilde{\Psi}_{\text{out}} = e^{-i\omega v_* + im\varphi + \lambda\theta_*}\, e^{2(i\omega - C_0)r_*}\, e^{i\pi C_0/\kappa}\, e^{\pi\omega/\kappa}, \quad (r < r_H). \tag{25}$$

So the relative scattering probability of the outgoing wave at the horizon is easily obtained

$$\left|\frac{\Psi_{\text{out}}}{\tilde{\Psi}_{\text{out}}}\right|^2 = e^{-2\pi\omega/\kappa}. \tag{26}$$

According to the method suggested by Damour and Ruffini (1976) and developed by Sannan (1988), the thermal radiation Fermionic spectrum of Dirac particles from the event horizon of the hole is given by

$$\langle N(\omega)\rangle = \frac{1}{e^{\omega/T_H} + 1}, \tag{27}$$

with the Hawking temperature being

$$T_H = \frac{\kappa}{2\pi},$$

whose obvious expression is

$$T_H = \frac{1}{4\pi r_H} \times \frac{Mr_H - r_H^3 a\ \cos\theta_0 + (2r_H a\ \cos\theta_0 - 1)(Q^2 + P^2) - \frac{\Lambda}{3} r_H^4 - r_{H,\theta}^2}{Mr_H + r_H^3 a\ \cos\theta_0 + (2r_H a\ \cos\theta_0 - 1/2)(Q^2 + P^2) - \frac{\Lambda}{6} r_H^4 + \frac{r_{H,\theta}^2}{2}}. \tag{28}$$

It follows that the temperature depends not only on the time, but also on the angle θ because it is determined by the surface gravity κ, a function of v and θ. The temperature is in consistence with that derived while investigating the thermal radiation of Klein–Gordon particles (Luo and Zhao, 1993; Zhao *et al.*, 1995; Zhu *et al.*, 1994a).

6. CONCLUSIONS

Equations (10) and (18) give the location and the temperature of event horizon of the hole, which depend not only on the advanced time v but also on the polar angle θ. Equation (27) shows the thermal radiation spectrum of Dirac particles in an arbitrarily rectilinearly accelerating Kinnersley black hole.

In conclusion, we have studied the Hawking radiation of Dirac particles in an arbitrarily accelerating Kinnersley black hole whose mass and charges change with time. The Chandrasekhar–Dirac equation cannot be decoupled in the most general black hole background, however, under the generalized tortoise coordinate transformation, the limiting form of its corresponding second-order equation takes the standard form of wave equation near the event horizon, to which separation of variables is possible.

Both the location and the temperature of the event horizon of the accelerating Kinnersley black hole depend on the time and the angle. They are just the same as those obtained in the discussion on thermal radiation of Klein–Gordon particles in the same space-time.

APPENDIX: NEWMAN–PENROSE COEFFICIENTS

The complex null-tetrad $\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}}\}$ that satisfies the orthogonal conditions $\boldsymbol{l} \cdot \boldsymbol{n} = -\boldsymbol{m} \cdot \bar{\boldsymbol{m}} = 1$ in the Kinnersley black hole is chosen as

$$\boldsymbol{l} = dv, \quad \boldsymbol{n} = G dv - dr - r^2 f \, d\theta,$$
$$\boldsymbol{m} = \frac{-r}{\sqrt{2}}(d\theta + i \sin\theta d\varphi), \qquad \bar{\boldsymbol{m}} = \frac{-r}{\sqrt{2}}(d\theta - i \sin\theta \, d\varphi). \tag{A1}$$

It is not difficult to determine the twelve Newman–Penrose complex coefficients (Newman and Penrose, 1962) in the above null-tetrad as follows

$$\tilde{\kappa} = \tilde{\lambda} = \sigma = \epsilon = 0, \quad \rho = \frac{1}{r}, \quad \mu = \frac{G}{r}, \quad \gamma = -G_{,r}/2, \quad \tau = -\tilde{\pi} = \frac{f}{\sqrt{2}},$$
$$\alpha = -\frac{\cot\theta}{2\sqrt{2}r} + \frac{f}{\sqrt{2}}, \quad \beta = \frac{\cot\theta}{2\sqrt{2}r}, \quad \nu = \frac{1}{\sqrt{2}r}[(2rG - r^2 G_{,r}) f + r^2 f_{,v} + G_{,\theta}].$$

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